ECS315 2017/1 Part VII Dr.Prapun

13 Three Types of Random Variables

13.1. Review: You may recall[57] the following properties for cdf of discrete random variables. These properties hold for any kind of random variables.

(a) The cdf is defined as $F_X(x) = P[X \leq x]$. This is valid for any type of random variables.

(b) Moreover, the cdf for any kind of random variable must satisfies three properties which we have discussed earlier:

CDF1 F_X is non-decreasing

CDF2 F_X is right-continuous

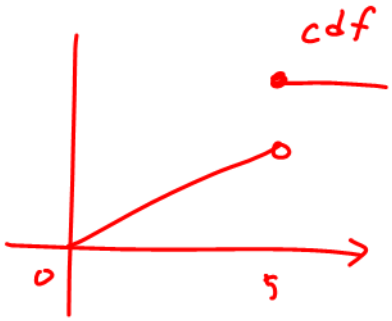
CDF3 $\lim_{x\to-\infty} F_X(x) = 0$ and $\lim_{x\to\infty} F_X(x) = 1$.

(c) $P[X = x] = F_X(x) - F_X(x^-)$ = the jump or saltus in F at x.

Theorem 13.2. If you find a function F that satisfies CDF1, CDF2, and CDF3 above, then F is a cdf of some random variable.

[57] If you don't know these properties by now, you should review them as soon as possible.

Example 13.3. Consider an input X to a device whose output Y will be the same as the input if the input level does not exceed 5. For input level that exceeds 5, the output will be saturated at 5. Suppose $X \sim \mathcal{U}(0, 6)$. Find $F_Y(y)$.



13.4. We can categorize random variables into three types according to its cdf:

(a) If $F_X(x)$ is piecewise flat with discontinuous jumps, then X is **discrete**.

(b) If $F_X(x)$ is a continuous function, then X is **continuous**.

(c) If $F_X(x)$ is a piecewise continuous function with discontinuities, then X is **mixed**.

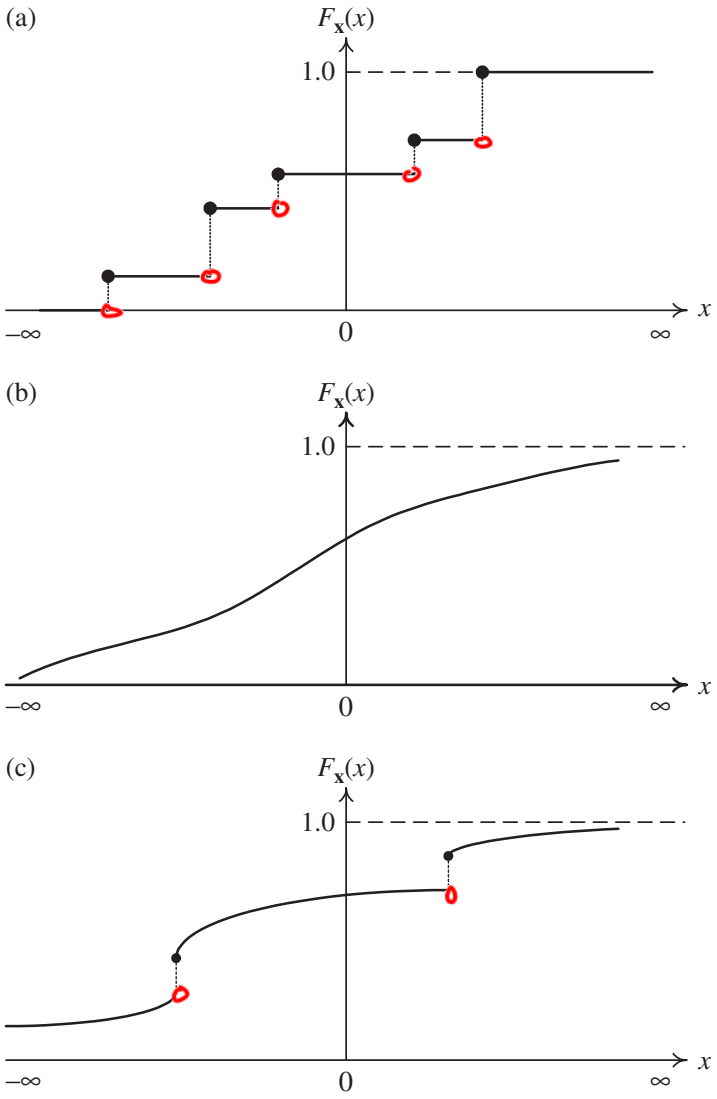


Figure 34: Typical cdfs: (a) a discrete random variable, (b) a continuous random variable, and (c) a mixed random variable [16, Fig. 3.2].